PMQ Final Focusing

using permanent magnets to build a "brute force" final focus for the muon collider

Outline

1. Approach to the problem
2. Permanent Magnet Quadrupoles
3. Triplet focusing for maximum IP demagnification
4. Implementation specifics at the Muon Collider

Muon Collider tentative parameters

√s (TeV)	1.5	3
Av. Luminosity / IP (10^{34}/cm^2/s)	0.8	3.4
Max. bending field (T)	10	14
Av. bending field in arcs (T)	6	8.4
Circumference (km)	3	4.5
No. of IPs	2	2
Repetition Rate (Hz)	15	12
Beam-beam parameter/IP	0.1	0.1
β* (cm)	1	0.5
Beam size @ IP (μm)	6	3
Bunch length (cm)	1	0.5
No. bunches / beam	1	1
No. muons/bunch (10^{12})	2	2
Norm. Trans. Emit. (μm)	25	25
Energy spread (%)	0.1	0.1
Norm. long. Emit. (m)	0.07	0.07
Total RF voltage (MV) at 800MHz	80	900
μ+ in collision / 8GeV proton	0.008	0.007
8 GeV proton beam power (MW)	4.8	4.3

Aperture of the focusing elements

$$\langle \mathcal{L} \rangle = f_0 \frac{n_b N_\mu^2}{4\pi \varepsilon_\perp \beta^*} h \times \frac{1}{2} \tau f_{rep} \sim \frac{P_\mu \xi}{C \beta^*} h \tau$$

P_μ – average muon beam power (~ γ)

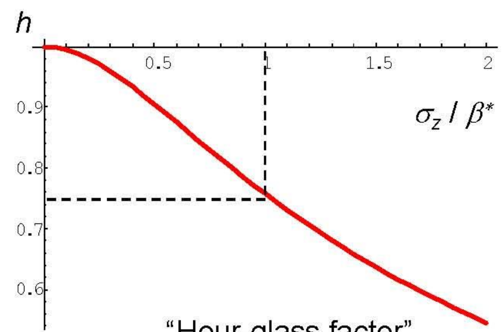
$$\xi = \frac{r_\mu N_\mu}{4\pi \gamma \varepsilon_\perp}$$ – beam-beam parameter

$\gamma\varepsilon_\perp$ – normalized emittance

C – collider circumference (~ γ if B=const)

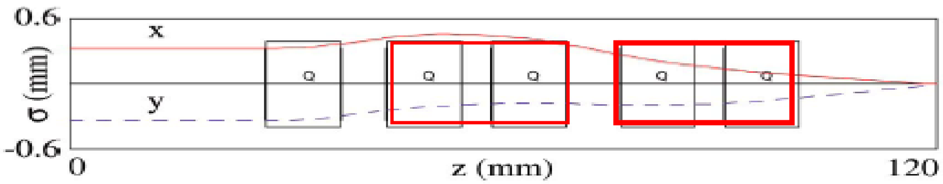
τ – muon lifetime (~ γ)

β^* – beta-function at IP



"Hour-glass factor"

UCLA's PMQs



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- Minimum focal length of the triplet set by detector equipment and triplet length
- Stronger gradient is better because it means a shorter magnet
- Analysis approach:
  1) What is the most powerful gradient that can be created?
  2) What will it mean for the final focus lattice and the beam itself?

To answer (1) we need…

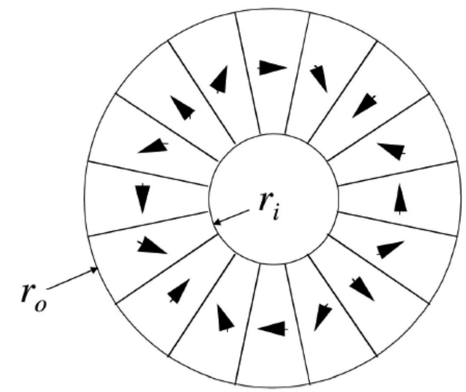
Halbach's PMQs

- Small inner radius
- Large remnant field
- a is the segmentation factor
  - 16 pieces a = 0.937
  - 32 pieces a = 0.984

5%!

$$B' = a\frac{2B_r}{r_i}\left(\frac{x-1}{x}\right)$$

$$r_o = x r_i$$



- Largest outer radius possible
  - x = 10 leaves 10% on the table
  - Engineering thick to thin can be mitigated with nesting

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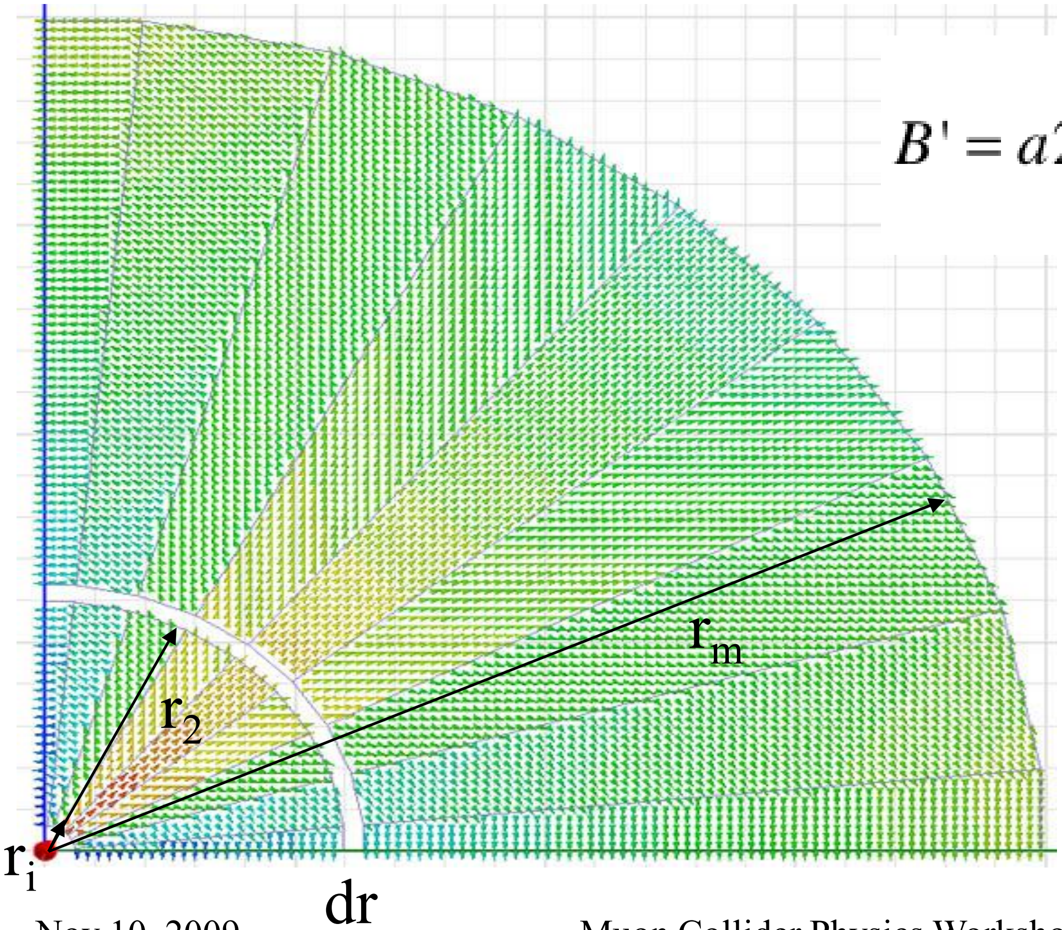
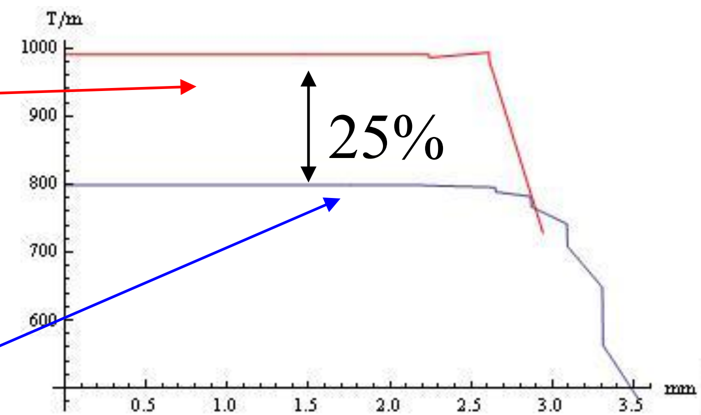
Nesting

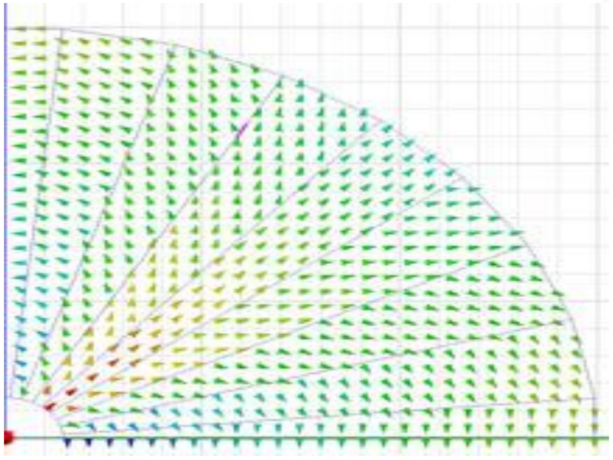


$$B' = a2B_r\left(\frac{1}{r_i} - \frac{1}{r_m} - \frac{\Delta r}{r_2(r_2 + \Delta r)}\right)$$

Examples

- r_i = 3 mm, x = 10, 32 segments (5 sigma beam)
- r_i = 3.6 mm, x = 10, 16 segments (6 sigma beam)
- $2\ B_r/r_i$ = 1130 T/m
  - B_r=1.7 T, r_i=3 mm





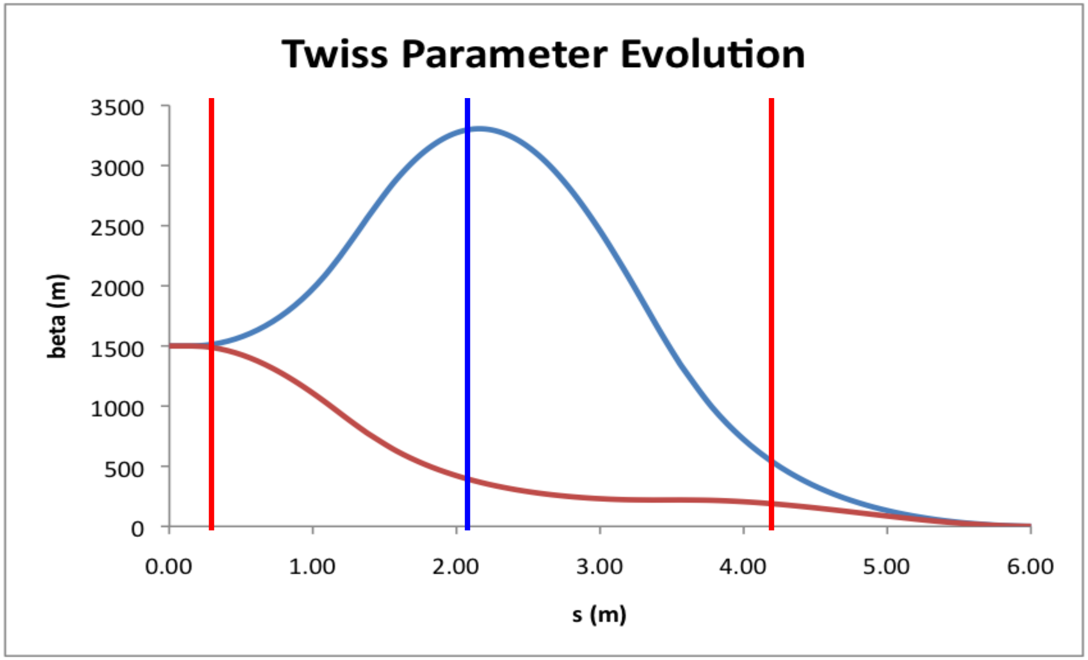
Maxwell 2D Results

Inner Radius -> # of Segments r_o=30 mm in both cases	5*sigma 3.0 mm	6*sigma 3.6 mm
16	960 T/m	780 T/m
32	990 T/m	810 T/m

First question answered!

Triplet Design I

- Assume: initial alpha_i = 0 and round beam
- Desire: round beam at waist at IP



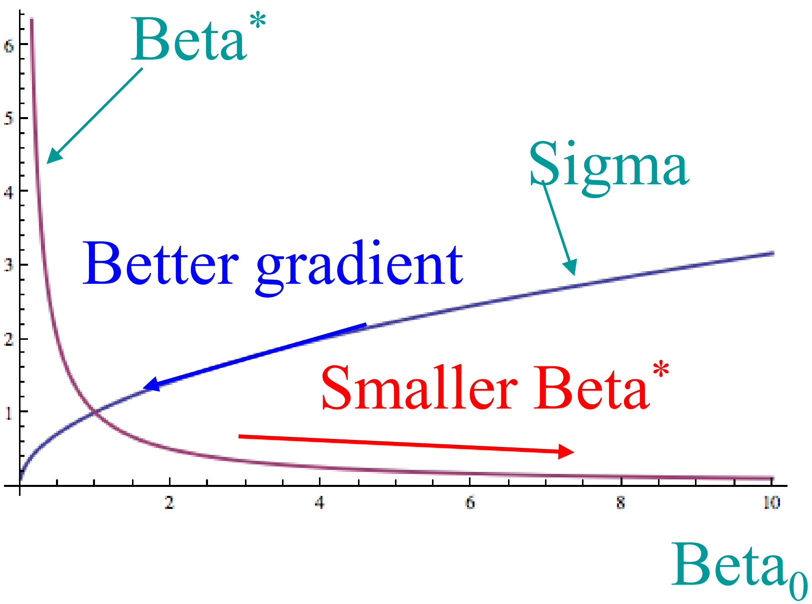
Triplet Design II

- Minimum beam size after a focusing system is a minimum when: $$\frac{\beta_0}{f_{eff}} \cong \frac{p}{\sigma_{\sigma_p}}$$
- Optimization of triplet leads to "obvious" solution of short focal length lenses with a large starting beta
- This leads to a problem…

$$\beta^* = \frac{4 f_Q^2}{\beta_0}$$

$$\alpha^* = \frac{2 f_Q}{\beta_0}$$

The trouble with beta and sigma



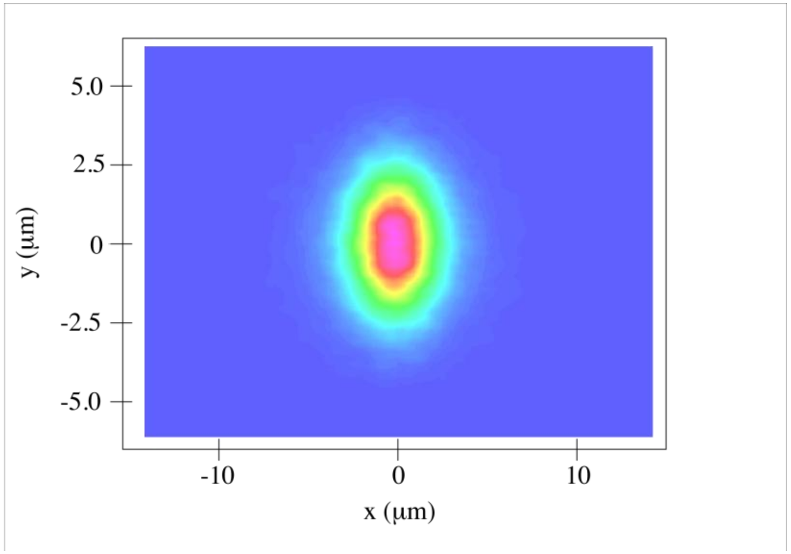
- Triplet design suggests large initial beta function while limited aperture requires small sigma

$$\sigma^2 = \frac{\beta \varepsilon_n}{\gamma}$$

Emittance vs Energy

$$\sigma^2 = \frac{\beta\varepsilon_n}{\gamma}$$

- The key to feasibility is shrinking emittance or dialing up energy
- The 2nd is unlikely so go with the 1st



How small must emittance be?

- Minimum beam size at IP when:

$$\frac{\beta_0}{f_{eff}} \cong \frac{p}{\sigma_{\sigma_p}}$$

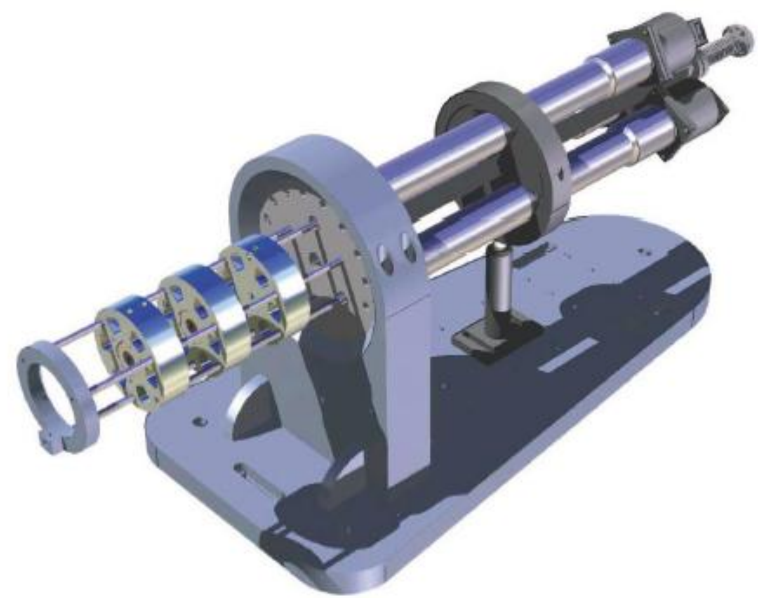
~1000 m

- Change beta and emittance without enlarging sigma
- With such difficult constraints why think about PMQs?

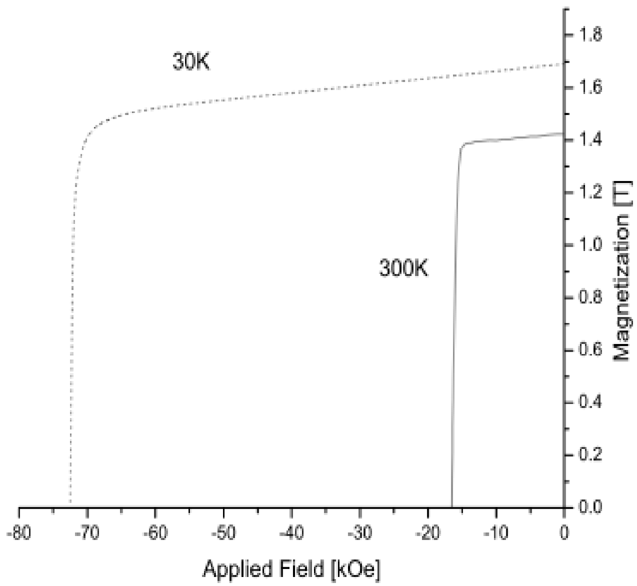
Beta (m)	Emit (micron)	B' (T/m)	Beta* (cm)
100	25	1000	17
700	3.6	1000	2.4
1500	1.7	1000	1.0
1500	1.7	750	1.5

Why PMQs?

- Opportunity to press higher field Praseodymium based cryogenic magnets into service
- Temperature tuning of gradient for cryogenic magnets?
- We have experience making them and using them for final focusing systems

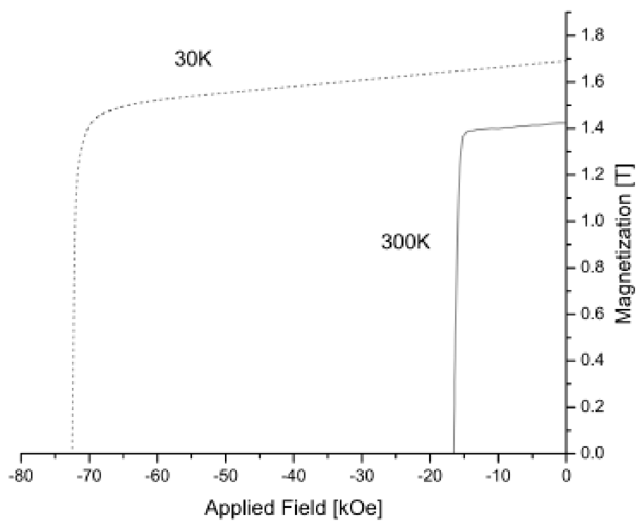


What’s so great about Pr?



- No spin axis reorientation like Neodymium
- Incredible coercivity when cooled
- Both H_{cj} and B_r increase with decreasing temperature
- Radiation resistant magnets are good for a collider with a decaying beam and near IP

Praseodymium Material



- Careful assembly and handling
- Local heating model of demagnetization predicts "bullet-proof" magnets with ample cooling power
- All studies on NdFeB magnets agree with this assessment
- At 30K $B_r = 1.7$ T

Conclusions

- Current goal of 20 pi mm -> 25 pi um must be extended to 2.5 pi um for a triplet PMQ solution that uses cutting edge materials
- "only" a factor of 10^4 reduction as compared to the current goal of 10^3

- Thanks to Gerard Andonian for the elegant simulations and lots of input